The Meaning of Beauty in the Exact Sciences

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When a representative of natural science is called upon to address a meeting of the Academy of Fine Arts, he can scarcely venture to express opinions on the subject of art; because the arts are certainly remote from his own field of activity. But perhaps be may be allowed to tackle the problem of beauty. For although the epithet "beautiful" (or "fine") is indeed employed here to characterize the arts, the realm of the beautiful stretches far beyond their territory. It assuredly encompasses other regions of mental life as well; and the beauty of nature is also reflected in the beauty of natural science.

Perhaps it will be best if, without any initial attempt at a philosophical analysis of the concept of "beauty," we simply ask where we can meet the beautiful in the sphere of exact science. Here I may perhaps be allowed to begin with a personal experience. When, as a small boy, I was attending the lowest classes of the Max-Gymnasium here in Munich, I became interested in numbers. It gave me pleasure to get to know their properties, to find out, for example, whether they were prime numbers or not, and to test whether they could perhaps be represented as sums of squares, or eventually to prove that there must be infinitely many primes. Now since my father thought my knowledge of Latin to be much more important than my numerical interests, he brought home to me one day from the National Library a treatise written in Latin by the mathematician Leopold Kronecker, in which the properties of whole numbers were set in relation to the geometrical problem of dividing a circle into a number of equal parts. How my father happened to light on this particular investigation from the middle of the last century I do not know, But the study of Kronecker's work made a deep impression on me. I sensed a quite immediate beauty in the fact that, from the problem of partitioning a

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circle, whose simplest cases were of course familiar to us in school, it was possible to learn something about the totally different sort of questions involved in elementary number theory. Far in the distance, no doubt, there already floated the question whether whole numbers and geometrical forms exist, i.e., whether they are there outside the human mind or whether they have merely been created by this mind as instruments for understanding the world. But at that time I was not yet able to think about such problems. The impression of something very beautiful was, however, perfectly direct; it required no justification or explanation.

But what was beautiful here? Even in antiquity there were two definitions of beauty, which stood in a certain opposition to one another. The controversy between them played a great part especially during the Renaissance. The one describes beauty as the proper conformity of the parts to one another, and to the whole. The other, stemming from Plotinus, describes it, without any reference to parts, as the translucence of the eternal splendor of the "one" through the material phenomenon. In our mathematical example we shall have to stop short, initially, at the first definition. The parts here are the properties of whole numbers and laws of geometrical constructions, while the whole is obviously the underlying system of mathematical axioms to which arithmetic and Euclidean geometry belong the great structure of interconnection guaranteed by the consistency of the axiom system. We perceive that the individual parts fit together, that as parts they do indeed belong to this whole, and without any reflection we feel the completeness and simplicity of this axiom system to be beautiful. Beauty is therefore involved with the age-old problem of the "one" and the "many" which occupied—in close connection with the problem of "being" and "becoming"—a central position in early Greek philosophy.

Since the roots of exact science are also to be found at this very point, it will be as well to retrace in broad outline the currents of thought in that early age. At the starting point of the Greek philosophy of nature there stands the question of a basic principle, from which the colorful variety of phenomena can be explained. However strangely it may strike us, the well-known answer of Thales—"Water is the material first principle of all things"—contains, according to Nietzsche, three basic philosophical demands which were to become important in the developments that followed: first, that one should seek for such a unitary basic principle; second, that the answer should be given only rationally, that is, not by reference to a myth; and third and finally, that in this context the material aspect of the world must play a deciding role. Behind these demands there stands, of course, the unspoken recognition that understanding can never mean anything more than the perception of connections, i.e., unitary features or marks of affinity in the manifold.

But if such a unitary principle of all things exists, then—and this was the next step along this line of thought—one is straight-way brought up against the question how it can serve to account for the fact of change. The difficulty is particularly apparent in the celebrated paradox of Parmenides. Only being is; non-being is not. But if only being is, there cannot be anything outside this being that articulates it or could bring about changes. Hence being will have to be conceived as eternal, uniform and unlimited in space and time. The changes we experience can thus be only an illusion.

Greek thought could not stay with this paradox for long. The eternal flux of appearances was immediately given, and the problem was to explain it. In attempting to overcome the difficulty, various philosophers struck out in different directions. One road led to the atomic theory of Democritus. In addition to being, non-being can still exist as a possibility, namely as the possibility for movement and form, or in other words, as empty space. Being is repeatable, and thus we arrive at the picture of atoms in the void—the picture that has since become infinitely fruitful as a foundation for natural science. But of this road we shall say no more just now. Our purpose, rather, is to present in more detail the other road, which led to Plato's Ideas, and which carries us directly into the problem of beauty.

This road begins in the school of Pythagoras. It is there that the notion is said to have originated that mathematics, the mathematical order, was the basic principle whereby the multiplicity of phenomena could be accounted for. Of Pythagoras himself we know little. His disciples seem, in fact, to have been a religious sect, and only the doctrine of transmigration and the laying down of certain moral and religious rules and prohibitions can be traced with any certainty to Pythagoras. But among these disciples—and this was what mattered subsequently—a preoccupation with music and mathematics played an important role. Here it was that Pythagoras is said to have made the famous discovery that vibrating strings under equal tension sound together in harmony if their lengths are in a simple numerical ratio. The mathematical structure, namely the numerical ratio as a source of harmony, was certainly one of the most momentous discoveries in the history of mankind. The harmonious concord of two strings yields a beautiful sound. Owing to the discomfort caused by beat-effects, the human ear finds dissonance disturbing, but consonance, the peace of harmony, it finds beautiful. Thus the mathematical relation was also the source of beauty.

Beauty, so the first of our ancient definitions ran, is the proper conformity of the parts to one another and to the whole. The parts here are the individual notes, while the whole is the harmonious sound. The mathematical relation can therefore assemble two initially independent parts into a whole, and so produce beauty. This discovery effected a breakthrough, in Pythagorean doctrine, to entirely new forms of thought, and so brought it about that the ultimate basis of all being was no longer envisaged as a sensory material such as water, in Thales—but as an ideal principle of form. This was to state a basic idea which later provided the foundation for all exact science. Aristotle, in his *Metaphysics*, reports that the Pythagoreans, "... who were the first to take up mathematics, not only advanced this study, but also having been brought up in it they thought its principles were the principles of all things... Since, again, they saw that the modifications and the ratios of the musical scales were expressible in numbers; since, then, all other things seemed in their whole nature to be modelled on numbers; and numbers seemed to be the first things in the whole of nature, they supposed the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and a number." (I, 5, 985b–986a; Ross's translation.)

Understanding of the colorful multiplicity of the phenomena was thus to conic about by recognizing in them unitary principles of form, which can be expressed in the language of mathematics. By this, too, a close connection was established between the intelligible and the beautiful. For if the beautiful is conceived as a conformity of the parts to one another and to the whole, and if, on the other hand, all understanding is first made possible by means of this formal connection, the experience of the beautiful becomes virtually identical with the experience of connections either understood or at least guessed at.

The next step along this road was taken by Plato, with the formulation of his theory of Ideas Plato contrasts the imperfect shapes of the corporeal world of the senses with the perfect forms of mathematics; the imperfectly circular orbits of the stars, say, with the perfection of the mathematically defined circle. Material things are the copies, the shadow images, of ideal shapes in reality; moreover, as we should be tempted to continue nowadays, these ideal shapes are actual because and insofar as they become "act"-ive in material events. Plato thus distinguishes here with complete clarity a corporeal being accessible to the senses and a purely ideal being apprehensible not by the senses but only through acts of mind. Nor is this ideal being in any way in need of man's thought in order to be brought forth by him. On the contrary, it is the true being, of which the corporeal world and human thinking are mere reproductions. As their name already indicates, the apprehension of Ideas by the human mind is more an artistic intuiting, a half-conscious intimation, than a knowledge conveyed by the understanding. It is a reminiscence of forms that were already implanted in this soul before its existence on earth. The central Idea is that of the Beautiful and the Good, in which the divine becomes visible and at sight of which the wings of the soul begin to grow. A passage in the Phaedrus (251 ff.) expresses the following thought: the soul is awe-stricken and shudders at the sight of the beautiful, for it feels that something is evoked in it that was not imparted to it from without by the senses but has always been already laid down there in a deeply unconscious region.

But let us conic back once more to understanding, and thus to natural science. The colorful multiplicity of the phenomena can be understood, according to Pythagoras and Plato, because and insofar as it is underlain by unitary principles of form susceptible of mathematical representation. This postulate already constitutes an anticipation of the entire program of contemporary exact science. It could not, however, be carried through in antiquity, since an empirical knowledge of the details of natural processes was largely lacking.

The first attempt to penetrate into these details was undertaken, as we know, in the philosophy of Aristotle. But in view of the infinite wealth initially presented here to the observing student of nature and the total lack of any sort of viewpoint from which an order might have been discernible, the unitary principles of form sought by Pythagoras and Plato were obliged to give place to the description of details. Thus there arose the conflict that has continued to this day in the debates, for example, between experimental and theoretical physics; the conflict between the empiricist, who by careful and scrupulous detailed investigation first furnishes the presuppositions for an understanding of nature, and the theoretician, who creates mathematical pictures whereby he seeks to order and so to understand nature—mathematical pictures that prove themselves, not only by their correct depiction of experience, but also and more especially by their simplicity and beauty, to be the true Ideas underlying the course of nature.

Aristotle himself, as an empiricist, was critical of the Pythagoreans, who, he said (*De Caelo*, II, 13, 293a), "are not seeking for theories and causes to account for observed facts, but rather forcing their observations and trying to accommodate them to certain theories and opinions of their own" and were thus setting up, one might say, as joint organizers of the universe. If we look back on the history of the exact sciences, it can perhaps be asserted that the correct representation of natural phenomena has evolved from this very tension between the two opposing views. Pure mathematical speculation becomes unfruitful because from playing with the wealth of possible forms it no longer finds its way back to the small number of forms according to which nature is actually constructed. And pure empiricism becomes unfruitful because it eventually bogs down in endless tabulation without inner connection. Only from the tension, the interplay between the wealth of facts and the mathematical forms that may possibly be appropriate to them, can decisive advances spring.

But in antiquity this tension was no longer acceptable, and thus the road to knowledge diverged for a long time from the road to the beautiful. The significance of the beautiful for the understanding of nature became clearly visible again only at the beginning of the modern period, once the way back had been found from Aristotle to Plato. And only through this change of course did the full fruitfulness become apparent of the mode of thought inaugurated by Pythagoras and Plato.

This is most clearly shown in the celebrated experiments on falling bodies that Galileo probably did not, in fact, conduct from the leaning tower of Pisa. Galileo begins with careful observations, paying no attention to the authority of Aristotle, but, following the teaching of Pythagoras and Plato, he does try to find mathematical forms corresponding to the facts obtained by experiment, and thus arrives at his laws of falling bodies. However, and this is a crucial point, he is obliged, in order to recognize the beauty of mathematical forms in the phenomena, to idealize the facts, or, as Aristotle disparagingly puts it, to force them. Aristotle had taught that all moving bodies not acted upon by external forces eventually come to rest, and this was the general experience. Galileo maintains, on the contrary, that in the absence of external forces bodies continue in a state of uniform motion. Galileo could venture to force the facts in this way because he could point out that moving bodies are of course always exposed to a frictional resistance, and that motion in fact continues the longer, the more effectively the frictional forces can be cut off. In exchange for this forcing of the facts, this idealization, he obtained a simple mathematical law, and this was the beginning of modern exact science.

Some years later, Kepler succeeded in discovering new mathematical forms in the data of his very careful observations of the planetary orbits, and in formulating the three famous laws that bear his name. How close Kepler felt himself in these discoveries to the ancient arguments of Pythagoras, and how much the beauty of the connections guided him in formulating them, can be seen from the fact that he compared the revolutions of the planets about the sun with the vibrations of a string, and spoke of a harmonious concord of the different planetary orbits, of a harmony of the spheres. At the end of his work on the harmony of the universe, he broke out into this cry of joy: "I thank thee, Lord God our Creator, that thou allowest me to see the beauty in thy work of creation." Kepler was profoundly struck by the fact that here he had chanced upon a central connection which had not been conceived by man, which it had been reserved to him to recognize for the first time–a connection of the highest beauty. A few decades later, Isaac Newton in England set forth this connection in all its completeness, and described it in detail in his great work Principia Mathematica. The road of exact science was thus pointed out in advance for almost two centuries.

But are we dealing here with knowledge merely, or also with the beautiful? And if the beautiful is also involved, what role did it play in the discovery of these connections? Let us again recall the first definition given in antiquity: "Beauty is the proper conformity of the parts to one another and to the whole." That this criterion applies in the highest degree to a structure like Newtonian mechanics is something that scarcely needs explaining. The parts are the individual mechanical processes—those which we carefully isolate by means of apparatus no less than those which occur inextricably before our eyes in the colorful play of phenomena. And the whole is the unitary principle of form which all these processes comply with and which was mathematically established by Newton in a simple system of axioms. Unity and simplicity are not, indeed, precisely the same. But the fact that in such a theory the many are confronted with the one, that in it the many arc unified, itself has the undoubted consequence that we also feel it at the same time to be simple and beautiful. The significance of the beautiful for the discovery of the true has at all times been recognized and emphasized. The Latin motto *"Simplex sigillum veri"*—"The simple is the seal of the true"—is inscribed in large letters in the physics auditorium of the University of Göttingen, as an admonition to those who would discover what is new; and another Latin motto, *"Pulchritudo splendor veritatis"*—"Beauty is the splendor of truth"—can also be interpreted to mean that the researcher first recognizes truth by this splendor, by the way it shines forth.

Twice more in the history of exact science, this shining forth of the great connection has been the crucial signal for a significant advance. I am thinking here of two events in the physics of our own century, the emergence of relativity theory and the quantum theory. In both cases, after years of vain effort at understanding, a bewildering plethora of details has been almost suddenly reduced to order by the appearance of a connection, largely unintuitable but still ultimately simple in its substance, that was immediately found convincing by virtue of its completeness and abstract beauty—convincing, that is, to all who could understand and speak such an abstract language.

But now, instead of pursuing the historical course of events any further, let us rather put the question quite directly: What is it that shines forth here? How comes it that with this shining forth of the beautiful into exact science the great connection becomes recognizable, even before it is understood in detail and before it can be rationally demonstrated? In what does the power of illumination consist, and what effect does it have on the onward progress of science?

Perhaps we should begin here by recalling a phenomenon that may be described as the unfolding of abstract structures. It can be illustrated by the example of number theory, which we referred to at the outset, but one may also point to comparable processes in the evolution of art. For the mathematical foundation of arithmetic, or the theory of numbers, a few simple axioms are sufficient, which in fact merely define exactly what counting is. But with these few axioms we have already posited that whole abundance of forms which has entered the minds of mathematicians only in the course of the long history of the subject—the theory of prime numbers, of quadratic residues, of numerical congruences, etc. One might say that the abstract structures posited in and with numbers have unfolded visibly only in the course of mathematical history, that they have generated the wealth of propositions and relationships that makes up the content of the complicated science of number theory. A similar position is also occupied—at the outset of an artistic style in architecture, say—by certain simple basic forms, such as the semicircle and rectangle in Romanesque architecture. From these basic forms there arise in the course of history new, more complicated and also altered forms, which yet can still in some way be regarded as variations on the same theme; and thus from the basic structures there emerges a new manner, a new style of building. We have the feeling, nonetheless, that the possibilities of development were already perceivable in these original forms, even at the outset; otherwise it would be scarcely comprehensible that many gifted artists should have so quickly resolved to pursue these new possibilities.

Such an unfolding of abstract basic structures has assuredly also occurred in the instances I have enumerated from the history of the exact sciences. This growth, this constant development of new branches, went on in Newtonian mechanics up to the middle of the last century. In relativity theory and the quantum theory we have experienced a similar development in the present century, and the growth has not yet conic to an end.

Moreover, in science as in art, this process also has an important social and ethical aspect; for many men can take an active part in it. When a great cathedral was to be built in the Middle Ages, many master masons and craftsmen were employed. They were imbued with the idea of beauty posited by the original forms, and were compelled by their task to carry out exact and meticulous work in accordance with these forms. In similar fashion, during the two centuries following Newton's discovery, many mathematicians, physicists and technicians were called upon to deal with specific mechanical problems according to the Newtonian methods, to carry out experiments or to effect technical applications; and here, too, extreme care was always required in order to attain what was possible within the framework of Newtonian mechanics. Perhaps it may be said in general that by means of the underlying structures, in this case Newtonian mechanics, guidelines were drawn or even standards of value set up, whereby it could be objectively decided whether a given task had been well or ill discharged. It is the very fact that specific requirements have been laid down, that the individual can assist by small contributions in the attainment of large goals and that the value of his contribution can be objectively determined, which gives rise to the satisfaction proceeding from such a development for the large number of people involved. Hence even the ethical significances of technology for our present age should not be underestimated.

The development of science and technology has also produced, for example, the Idea of the airplane. The individual technician who assembles some component for such a plane, the artisan who makes it, knows that his work calls for the utmost care and exactitude and that the lives of many may well depend upon its reliability. Hence he can take pride in a well-executed piece of work, and delights, as we do, in the beauty of the aircraft, when he feels that in it the technical goal has been realized by properly adequate means. Beauty, so runs the ancient definition we have already often cited, is the proper conformity of the parts to one another and to the whole, and this requirement must also be satisfied in a good aircraft.

But in pointing thus to the evolution of beauty's ground structure, to the ethical values and demands that subsequently emerge in the historical course of development, we have not yet answered the question we asked earlier, namely, what it is that shines forth in these structures, how the great connection is recognized even before it is rationally understood in detail. Here we ought to reckon in advance with the possibility that even such recognition may be founded upon illusions. But it cannot be doubted that there actually is this perfectly immediate recognition, this shuddering before the beautiful, of which Plato speaks in the *Phaedrus*.

Among all those who have pondered on this question, it seems to have been universally agreed that this immediate recognition is not a consequence of discursive (i.e., rational) thinking. I should like here to cite two statements, one from Johannes Kepler, who has already been referred to, and the other, in our own time, from the Zürich atomic physicist Wolfgang Pauli, who was a friend of the psychologist, Carl Jung. The first passage is to be found in Kepler's *Harmony of the World*:

> That faculty which perceives and recognizes the noble proportions in what is given to the senses, and in other things situated outside itself, must be ascribed to the lower region of the soul. It lies very close to the faculty which supplies formal schemata to the senses, or deeper still, and thus adjacent to the purely vital power of the soul, which does not think discursively, i.e., in conclusions, as the philosophers do, and employs no considered method, and is thus not peculiar only to man, but also dwells in wild animals and the dear beasts of the field... Now it might be asked how this faculty of the soul, which does not engage in conceptual thinking, and can therefore have no proper knowledge of harmonic relations, should be capable of recognizing what is given in the outside world. For to recognize is to compare the sense perception outside with the original pictures inside, and to judge that it conforms to them. Proclus has expressed the matter very finely in his simile of awakening, as from a dream. For just as the sensorily presented things in the outer world recall to us those which we formerly perceived in the dream, so also the mathematical relations given in sensibility call forth those intelligible archetypes which were already given inwardly beforehand, so that they now shine forth truly and vividly in the soul, where before they were only obscurely present there. But how have they come

> to be within? To this I answer that all pure Ideas or archetypal patterns of harmony, such as we were speaking of, are inherently present in those who are capable of apprehending them. But they are not first received into the mind by a conceptual process, being the product, rather, of a sort of instinctive intuition of pure quantity, and are innate in these individuals, just as the number of petals in a plant, say, is innate in its form principle, or the number of its seed chambers is innate in the apple.

So far Kepler. He is therefore referring us here to possibilities already to be found in the animal and plant kingdoms, to innate archetypes that bring about the recognition of forms.

In our own day, Adolf Portmann, in particular, has described such possibilities, pointing for example to specific color patterns seen in the plumage of birds, which can possess a biological meaning only if they are also perceived by other members of the same species. The perceptual capacity will therefore have to be just as innate as the pattern itself. We may also consider bird song at this point. At first the biological requirement here may well have been simply for a specific acoustic signal, serving to seek out the partner and understood by the latter. But to the extent that this immediate biological function declines in importance, a playful enlargement of the stock of forms may ensue, an unfolding of the underlying melodic structure, which is then found enchanting as song by even so alien a species as man. The capacity to recognize this play of forms must at all events be innate to the species of bird in question, for certainly it has no need of discursive, rational thought. In man, to cite another example, there is probably an inborn capacity for understanding certain basic forms of the language of gesture, and thus for deciding, say, whether the other has friendly or hostile intentions—a capacity of the utmost importance for man's communal life.

Ideas similar to those of Kepler have been put forward in an essay by Pauli. He writes:

> The process of understanding in nature, together with the joy that man feels in understanding, i.e., in becoming acquainted with new knowledge, seems therefore to rest upon a correspondence, a coming into congruence of preexistent internal images of the human psyche with external objects and their behavior. This view of natural knowledge goes back, of course, to Plato and was ... also very plainly adopted by Kepler. The latter speaks, in fact, of Ideas, preexistent in the mind of God and imprinted accordingly upon the soul, as the image of God. These primal images, which the soul can perceive by means of an innate instinct, Kepler calls archetypes. There is very wide-ranging agreement here with the primordial images or archetypes introduced into modern psychology

> by C. G. Jung, which function as instinctive patterns of ideation. In that modern psychology has given proof that all understanding is a protracted affair, accompanied by processes in the unconscious long before the content of consciousness can be rationally formulated, it has again directed attention to the preconscious, archaic stage of cognition. At this stage, the place of clear concepts is taken by images of strongly emotional content, which are not thought but are seen pictorially, as it were, before the mind's eye. Insofar as these images are the expression of a suspected but still unknown state of affairs, they can also be called symbolic, according to the definition of a symbol proposed by Jung. As ordering operators and formatives in this world of symbolic images, the archetypes function, indeed, as the desired bridge between sense perceptions and Ideas, and are therefore also a necessary precondition for the emergence of a scientific theory. Yet one must beware of displacing this a priori of knowledge into consciousness, and relating it to specific, rationally formulable Ideas.

In the further course of his inquiries, Pauli then goes on to show that Kepler did not derive his conviction of the correctness of the Copernican system primarily from any particular data of astronomical observation but rather from the agreement of the Copernican picture with an archetype which Jung calls a *mandala*, and which was also used by Kepler as a symbol for the Trinity. God, as prime mover, is seen at the center of a sphere; the world, in which the Son works, is compared with the sphere's surface; and the Holy Ghost corresponds to the beams that radiate from center to surface of the sphere. It is naturally characteristic of these primal images that they cannot really be rationally or even intuitively described.

Although Kepler may have acquired his conviction of the correctness of Copernicanism from primal images of this kind, it remains a crucial precondition for any usable scientific theory that it should subsequently stand up to empirical testing and rational analysis. In this respect the sciences are in a happier position than the arts, since for science there is an inexorable and irrevocable criterion of value that no piece of work can evade. The Copernican system, the Keplerian laws and the Newtonian mechanics have subsequently proved themselves in the interpreting of phenomena, in observational findings and in technology, over such a range and with such extreme accuracy that after Newton's *Principia* it was no longer possible to doubt that they were correct. Yet even here there was still an idealization involved, such as Plato had held necessary and Aristotle had disapproved.

This only came out in full clarity some fifty years ago, when it was realized from the findings in atomic physics that the Newtonian scheme of concepts was no longer adequate to cope with the mechanical phenomena in the interior of the atom. Since Planck's discovery of the quantum of action, in 1900, a state of confusion had arisen in physics. The old rules, whereby nature had been successfully described for more than two centuries, would no longer fit the new findings. But even these findings were themselves inherently contradictory. A hypothesis that proved itself in one experiment failed in another. The beauty and completeness of the old physics seemed destroyed, without anyone having been able, from the often disparate experiments, to gain a real insight into new and different sorts of connection. I don't know if it is fitting to compare the state of physics in those twenty-five years after Planck's discovery (which I too encountered as a young student) to the circumstances of contemporary modern art. But I have to confess that this comparison repeatedly comes to my mind. The helplessness when faced with the question of what to do about the bewildering phenomena, the lamenting over lost connections, which still continue to look so very convincing—all these discontents have shaped the face of both disciplines and both periods, different as they are, in a similar manner. We are obviously concerned here with a necessary intervening stage, which cannot be by-passed and which is preparing for developments to come. For as Pauli told us, all understanding is a protracted affair, inaugurated by processes in the unconscious long before the content of consciousness can be rationally formulated. The archetypes function as the desired bridge between the sense perceptions and the Ideas.

At that moment, however, when the true Ideas rise up, there occurs in the soul of him who sees them an altogether indescribable process of the highest intensity. It is the amazed awe that Plato speaks of in the *Phaedrus*, with which the soul remembers, as it were, something it had unconsciously possessed all along. Kepler says: *"Geometria est archetypus pulchritudinis mundi"*; or, if we may translate in more general terms—"Mathematics is the archetype of the beauty of the world." In atomic physics this process took place not quite fifty years ago, and has again restored exact science, under entirely new presuppositions, to that state of harmonious completeness which for a quarter of a century it had lost. I see no reason why the same thing should not also happen one day in art. But it must be added, by way of warning, that such a thing cannot be made to happen–it has to occur on its own.

Ladies and gentlemen, I have set this aspect of exact science before you because in it the affinity with the fine arts becomes most plainly visible, and because here one may counter the misapprehension that natural science and technology are concerned solely with precise observation and rational, discursive thought. To be sure, this rational thinking and careful measurement belong to the scientist's work, just as the hammer and chisel belong to the work of the sculptor. But in both cases they are merely the tools and not the content of the work.

Perhaps at the very end I may remind you once more of the second definition of the concept of beauty, which stems from Plotinus and in which no more is heard of the parts and the whole: "Beauty is the translucence, through the material phenomenon, of the eternal splendor of the 'one.'" There are important periods of art in which this definition is more appropriate than the first, and to such periods we often look longingly back. But in our own time it is hard to speak of beauty from this aspect, and perhaps it is a good rule to adhere to the custom of the age one has to live in, and to keep silent about that which it is difficult to say. In actual fact the two definitions are not so very widely removed from one another. So let us be content with the first and more sober definition of beauty, which certainly is also realized in natural science; and let us declare that in exact science, no less than in the arts, it is the most important source of illumination and clarity.